Pre-class Warm-up !!!

Select the correct answer:

A solution to the differential equation
$dy/dx = x - y$ is

a. a number, like 5.

b. a pair of numbers, like (5,7)

c. a function $y = f(x)$ ✓

d. a function $g(x,y)$

e. None of the above.

Section 1.2: integrals as solutions

New vocabulary:

- general solution
- Particular solution to an IVP = initial value problem = d.e. + initial condition

1.2 question 4:

Solve $\frac{dy}{dx} = \frac{1}{x^2}$, $y(1) = 5$.

Solution: $y = -\frac{1}{x} + C$ is the general solution.

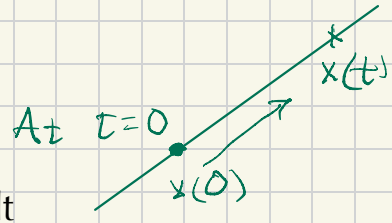
$y(1) = -\frac{1}{1} + C = 5$

$C = 5 + 1 = 6$.

$y = -\frac{1}{x} + 6$.

Motion in a straight line

Position x(t)
Velocity v(t) = dx/dt
Acceleration a(t) = dv/dt



Thus x is an antiderivative of v, and v is an antiderivative of a.
Some questions (13-18) have variable a(t).
When acceleration is constant a(t) = a,
formula (11) on page 12 says:

$$x(t) = \frac{1}{2}at^2 + v_0(t) + x_0$$

where $v_0 = v(0)$, $x_0 = x(0)$.

Proof. If $a(t) = a$ then

$a = \frac{dv}{dt}$ so $v = at + C$.

Put $t = 0$ to get $v_0 = C$

$v = at + v_0 = \frac{dx}{dt}$

Thus $x = \frac{1}{2}at^2 + v_0 t + D$

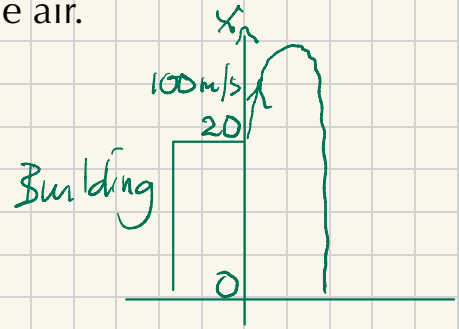
Put $t = 0$ to get $D = x_0$

$x(t) = \frac{1}{2}at^2 + v_0 t + x_0$.

We can take the acceleration due to gravity to be 32 ft/s^2 or about 10 m/s^2

1.2 question 26:
A projectile is fired straight up at 100 m/s from the top of a building 20 m high and falls to the ground at the base of the building. Find
(a) the maximum height above the ground.
(b) when it passes the top of the building,
(c) the total time in the air.

Solution:



$a = 10 m/s^2$

a) Solve $v = \frac{dx}{dt} = 100 - 10t = 0$

$t = \frac{100}{10} = 10$, $x(10) = 20 + 1000 - 500 = 520$

b) Solve $x(t) = 20 = 20 + 100t - 5t^2$

$5t^2 - 100t = 5t(t-20) = 0$

$t = 0$ or 20. We want $t = 20$.

Question: which is the correct equation of motion?

a. x(t) = 20 + 100t +10t^2
b. x(t) = 20 + 100t + 5t^2
c. x(t) = 20 + 100t - 5t^2 ✓
d. x(t) = 20 + 100t - 10t^2

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c) Solve $x(t) = 0$

$= 20 + 100t - 5t^2$.

$= \frac{20 + \sqrt{416}}{2}$.

$(20+\sqrt{416})/2$

1.2 question 30:
A car at 60 mph = 88 f/s skids to a stop in 176 feet. Assuming constant deceleration, what is the deceleration? How long did the skid continue?

$x(t) = 88t + at^2 / 2$

$v = 88 + at$

$v = 0$ when $t = -88/a$

$176 = 88(-88/a) + a(88^2/a^2)/2$

Solve. Then use $t = -88/a$